Operation of the Field Displacement Isolator in Rectangular Waveguide*

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Summary—**A field displacement isolator in WR-159 rectangular waveguide consisting of a full height ferrite slab having a resistive film on one face is treated analytically. The resultant transcendental equation was programmed for a computer and values of the propagation constant found in the frequency range 5.90 to 6.45 kMc for various film resistivities. Two TE modes are found to exist whose relative behavior depends on the resistivity of the film.**

Reasonably close experimental verification of the results was obtained for the total attenuation and for the predicted *E*-field distributions by *E*-field probe tests. Additional attenuation above that predicted by the theory for a single mode is observed as a result of an interference at the end of the ferrite.

A partial height ferrite slab isolator was subjected to *E*-field probe tests. The field distributions were found to be similar to the full height case. Here, also, additional attenuation is obtained at some frequencies as a result of an interference.

Introduction

THE field displacement isolator has proved to be a valuable device in microwave transmission systems where low attenuation in the forward direction of transmission and high attenuation in the reverse direction are desired. In rectangular waveguide it has usually taken the form of an off-center slab of ferrite having a resistance sheet on the side nearer the center of the waveguide as shown in Fig. 1(a). The best performance so far has been obtained with "partial height" ferrites, where there is an air gap between all faces of the ferrite and the waveguide walls [Fig. 1(b)].

This type of isolator has been made to have reverse to forward attenuation ratios as high as 150 to 1 db over an 8 per cent bandwidth at 6 kMc. Such a device was first described by Fox, Miller, and Weiss.[1] A more complete treatment was given by Weisbaum and Seidel.[2] More recently, Button[3] has given an analysis which is quite useful in a preliminary determination of the thickness of the ferrite slab and its spacing from the sidewall for the "full height" case of Fig. 1(a). Other authors have treated the problem of propagation in ferrite-loaded waveguide but none has, to the best of our knowledge, considered the case of the field displacement isolator with resistance sheet, or the case of the "partial height" ferrite. The "partial height" case would be very difficult to treat analytically.

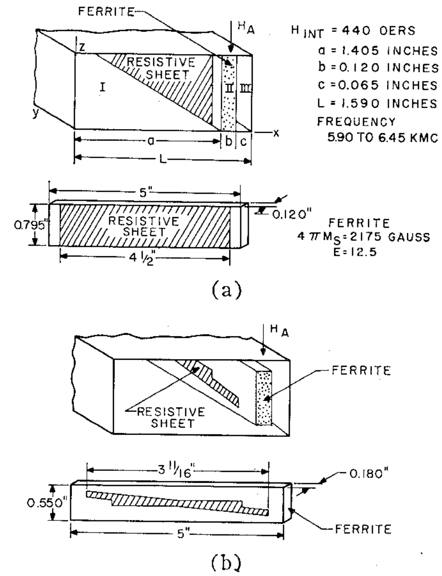

Fig. 1—Field displacement isolator configurations and dimensions. (a) Full height isolator. (b) Partial height isolator.

W. J. Crowe[4] has calculated modes in ferrite-loaded rectangular waveguide with consideration of the effect of magnetic losses in the ferrite. We have treated a similar problem in which magnetic loss is neglected and a resistive sheet is introduced on the face of the ferrite.

Analysis of the "full height" case in WR-159 waveguide has been carried out for different loss film resistivities over the 500-Mc frequency band centered at 6.175 kMc.

The experimental work has consisted of measurement of attenuation and electric field patterns of isolators of both "full height" and "partial height" configurations. A quite powerful experimental tool for this work is the *E*-field probe. One of the authors[5] had considerable experience in its use prior to this work. More recently, other papers dealing with field probe experiments on ferrite-loaded waveguide have appeared.[6,7] One of these[7]

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[1] A. G. Fox, S. E. Miller and M. T. Weiss, "Behavior and applications of ferrites in the microwave region," *Bell Syst. Tech. J.*, vol. 34, pp. 5–103; January, 1955.

[2] S. Weisbaum and H. Seidel, "The field displacement isolator," *Bell Syst. Tech. J.*, vol. 35, pp. 877–898; July, 1956.

[3] K. J. Button, "Theoretical analysis of the operation of the field displacement ferrite isolator," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 303–308; July, 1958.

[4] W. J. Crowe, "Behavior of TE modes in ferrite loaded rectangular waveguide in the region of ferromagnetic resonance," *J. Appl. Phys.*, vol. 29, pp. 397–398; March, 1958.

[5] R. L. Comstock, D. J. Angelakos and A. Johnson, "Determination of Fields in Ferrite Loaded Waveguide," University of California, Berkeley, Engrg. Res. Ser. No. 60, Issue No. 186; July 16, 1957.

[6] D. J. Angelakos, "Transverse electric field distributions in ferrite loaded waveguides," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 390–391; July, 1959.

[7] T. M. Straus, "Field Displacement Effect in Dielectric and Ferrite Loaded Waveguides," 1958 IRE WESCON CONVENTION RECORD, pt. I; pp. 135–145.

gives some data on a field displacement isolator which are qualitatively consistent with some of our results.

The ferrite used in this experimental work was General Ceramics Corporation's Ferramic R1, a magnesium manganese ferrite having a saturation magnetization of 2175 gauss. This had been found to make good isolators at 6 kMc. In preliminary experiments the thickness of the ferrite slab and its position in the waveguide had been varied until a situation was achieved where the minimum forward loss and a reasonably high reverse loss occurred at the same applied biasing field at a frequency of 6.175 kMc. This was done to provide a starting point for theoretical and experimental work which would be near a useful operating condition. The dimensions chosen for the ferrite bar were: length 5 inches, width 0.795 inch (equal to the narrow dimension of the waveguide), thickness 0.120 inch.

The waveguide used was WR-159 rectangular waveguide having transverse internal dimensions 1.590 inches by 0.795 inch, and the spacing of the ferrite from the waveguide sidewall was 0.065 inch.

Theoretical Considerations

The Boundary Value Problem

It is desired to find the solution to the boundary value problem of a lossless ferrite slab, with a resistance sheet on one face, in rectangular TE mode waveguide [Fig. 1(a)]. The electric fields in the three regions shown in the figure are given by the following expressions:

Region	*Electric Field*
I	$E_z = \mathcal{A} \sin (k_a x) e^{-\Gamma y}$
II	$E_z = [\mathcal{B} e^{jk_m x} + \mathcal{C}_e^{-jk_m x}] e^{-\Gamma y}$
III	$E_z = \mathcal{D} \sin [k_a(L - x)] e^{-\Gamma y}$

where $\Gamma = \alpha + j\beta$ is the complex propagation constant. The transverse wave numbers are:

Region	*Transverse Wave Number*
I, III	$k_a = \sqrt{\frac{\omega^2}{c^2} + \Gamma^2}$
II	$k_m = \sqrt{\frac{\omega^2}{c^2} \epsilon \mu_e + \Gamma^2}$

where ϵ is the relative dielectric constant and μ_e is the effective permeability, or $\mu_e = (\mu^2 - k^2)/\mu$, in which μ and k are the diagonal and off-diagonal components of the Polder tensor[8] which describes the ferrite permeability. To evaluate Γ it is not necessary to solve the boundary value problem completely and evaluate the constants $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$. H. Seidel[9] has developed a method for the determination of Γ for structures of this type. Briefly, the method consists in representing the ferrite slab, resistance sheet, empty waveguide and any other region in the laminated waveguide cross-section by a transverse matrix operator. The particular operator chosen was the *ABCD* matrix which transfers the vector

$$\begin{bmatrix} E_z \\ H_y \end{bmatrix}$$

from one vertical longitudinal plane to another, *i.e.*,

$$\begin{bmatrix} E_z \\ H_y \end{bmatrix}_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_z \\ H_y \end{bmatrix}_2$$

where subscripts 1 and 2 denote longitudinal planes in the waveguide.

Once the operators for each region are determined, they are multiplied to give an over-all matrix for the structure. From the above relation we obtain $E_{z_1} = AE_{z_2} + BH_{y_2}$. If we choose planes 1 and 2 to be the waveguide sidewalls, then $E_{z_1} = E_{z_2} = 0$. This requires that $B \equiv 0$. Putting $B = 0$ results in a transcendental equation from which the propagation constant Γ can be found as will be shown.

The relation involving the individual operators is as follows:

$$\begin{bmatrix} E_z \\ H_y \end{bmatrix}_1 = \underset{\text{(region I)}}{\begin{bmatrix} \cos \Phi & j \sin \Phi \\ j \sin \Phi & \cos \Phi \end{bmatrix}} \underset{\text{(resistance sheet)}}{\begin{bmatrix} 1 & 0 \\ \frac{1}{Z'} & 1 \end{bmatrix}}$$

$$\cdot \underset{\text{(ferrite—region II)}}{\begin{bmatrix} \cos \theta - \nu \sin \theta & j\zeta \sin \theta \\ \frac{j}{\zeta}(1 + \nu^2) \sin \theta & \cos \theta + \nu \sin \theta \end{bmatrix}}$$

$$\cdot \underset{\text{(region III)}}{\begin{bmatrix} \cos \psi & j \sin \psi \\ j \sin \psi & \cos \psi \end{bmatrix}} \begin{bmatrix} E_z \\ H_y \end{bmatrix}_2, \tag{1}$$

where:

$$\Phi = k_a a$$

$$\theta = k_m b$$

$$\psi = k_a c$$

$$\nu = \frac{-jk\Gamma}{\mu k_m}$$

$$\zeta = \frac{k_a}{k_m} \frac{\mu^2 - k^2}{\mu}$$

$$Z' = \frac{R_S k_a}{\omega \mu_0}.$$

[8] D. Polder, "On the theory of ferromagnetic resonance," *Phil. Mag.*, vol. 40, pp. 99–115; January, 1949.

[9] H. Seidel, "Ferrite slabs in transverse electric mode waveguide," *J. Appl. Phys.*, vol. 28, pp. 218–226; February, 1957.

Performing the indicated multiplication and setting the B term of the over-all matrix equal to zero results in the following equation:

$$\sin(\psi+\Phi)+\frac{j\sin\Psi\sin\Phi}{Z'}$$
$$+\nu\tan\theta\left[\sin(\Phi-\psi)-\frac{j}{Z'}\sin\psi\sin\Phi\right]$$
$$+\tan\theta\left[\zeta\cos\psi\cos\Phi+\frac{j\zeta}{Z'}\cos\psi\sin\Phi\right.$$
$$\left.-\frac{\sin\Phi\sin\psi}{\zeta}(1+\nu^2)\right]=0. \quad (2)$$

An IBM 704 electronic data processing machine was used to find the roots of (2).

Solutions of the Characteristic Equation

The first set of solutions of (2) was obtained for the case studied by Button,[3] that is, with no resistance sheet ($R_S=\infty$). The ferrite and its position in the waveguide used in the calculations are described in the introduction and all calculations use the parameters given there. The solutions for this case are shown by the solid curves in Fig. 2. The propagation constants $\beta+$ and $\beta-$ are given as functions of the internal applied biasing field, H_A, for a frequency of 6.175 kMc. It is seen that there are two propagating modes. The one labeled "TE_{10}" is the dominant mode and is the one discussed by Button and others. When H_A is less than 215 oersteds, another mode labeled "TE_{20}" can propagate. These modes have the general properties of their empty waveguide equivalents but are in this case distorted by the asymmetrically placed ferrite slab. The quotation marks are used to indicate the distorted modes in the ferrite-loaded waveguide as distinguished from the empty waveguide modes of similar designation. It should be noted that the problem of determining the proportion of the energy scattered into each of these modes was not attempted. However, experimental work has shown that generally both will be excited in significant amounts. As soon as appreciable loss is introduced, the situation in regard to TE_{20} changes markedly. The dotted curve of Fig. 2 shows that with R_S of 5000 ohms per square, a relatively high value, the TE_{20} mode can now propagate over the entire range of applied field shown. The solutions for the "forward" direction ($\beta+$) are seen to remain essentially independent of H_A over the interval shown, a conclusion also reached by Weisbaum and Seidel.[2] These solutions correspond closely to the value of β (and hence k_a) required by $k_a a=\pi$ for the condition that the electric field is zero at the face of the ferrite. The computed value of H_A which results in a true null at the face of the ferrite for this frequency is 440 oersteds (internal biasing field).

Fig. 3 is a plot of solutions to (2) for a large range of resistivities of the loss film. Values of α and β are shown for a biasing field of 440 oersteds and a frequency of 6.175 kMc. These are for the "reverse" or high attenuation direction of propagation under the same conditions that produced the null at the face of the ferrite and hence theoretically no loss for the "forward" direction. For extremely low values of R_S the TE_{10} mode is highly attenuated and all propagation is in the TE_{20} mode or possibly higher modes. For $R_S=0$ the system is just a waveguide of width a (Fig. 1), and is reciprocal. For all values of R_S, at least one of the modes propagates with a value of $\alpha\leq 8.6$ db per inch (Fig. 3). This then represents the maximum attenuation to be expected for this bias and frequency since, as will be shown in the experimental results, the energy seems to propagate principally in the mode with the lowest loss.

Fig. 4 is a plot in the complex Γ plane of solution curves with frequency as a parameter. The solutions are given for three representative values of resistivity: $R_S=670$, 1000 and 2500 ohms per square. Several things should be noted about the character of these solutions.

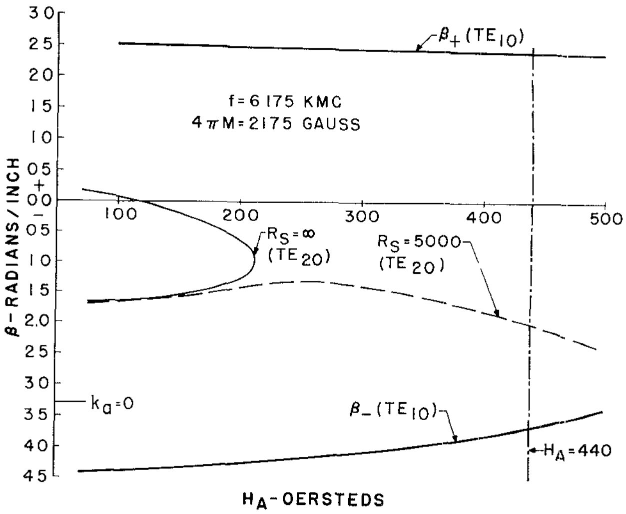

Fig. 2—Propagation constants for ferrite slab-loaded waveguide of Fig. 1(a).

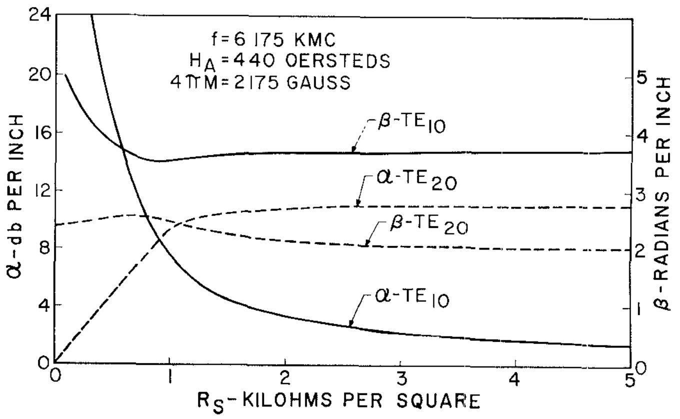

Fig. 3—Propagation constants for "reverse" direction with resistive sheet of the configuration shown in Fig. 1(a).

First, in contrast to unloaded waveguide solutions, it is difficult to determine when a given mode is truly cut off. The reason for this is inherent in problems involving waveguides loaded with ferromagnetic media—the nonreciprocity of the medium is manifested in the characteristic equations by a linear term in Γ. Because of the presence of this term, the propagation constants of the modes, which in the absence of loss would be cut off, are complex. However, if a given mode has a large value of α, *e.g.*, $\alpha > 15$ db per inch, the mode can be regarded as nonpropagating regardless of the value of β.

A second point of interest regarding these curves is the ambiguity in labeling them consistently. For example, if the resistivity of the loss sheet were increased from 670 ohms per square, at the lower frequencies the TE_{20} mode would change into a TE_{10} mode, while at the higher frequencies it would remain a TE_{20} mode. W. J. Crowe[4] has discussed this problem in relation to the TE mode solutions in magnetically lossy slabs in waveguides and has shown that it results from the existence of singular points at which the left side of the characteristic equation, *e.g.*, (2), and its partial derivative with respect to Γ, are zero simultaneously. Such a point is observed in Fig. 4 and is labeled "A". At this point the two modes become degenerate at one frequency and changes in resistivity near this critical point result in the TE_{10} mode becoming a TE_{20} mode and vice versa.

The case of the double slab isolator having two ferrite slabs symmetrically disposed with respect to the center of the waveguide but having oppositely polarized biasing fields was not considered. One can see intuitively that the field distribution for this case must be either even or odd with respect to the center plane of the waveguide. Advantage of this could be taken in the analysis in that it would only be necessary to use the half-circuit transverse matrix operator. Lax, Button and Roth[10] have considered this structure without resistive sheets on the ferrites and have found the expected symmetries in the field configurations. Weisbaum and Boyet[11] have shown results on a partial height isolator of this type built for a commercial application.

Experimental Results

Apparatus and Procedure

The *E*-field probe used in these experiments was a transverse traveling probe inserted in a broad face of a length of WR-159 waveguide. The probe itself was a fine wire extending about 3/32 inch into the waveguide. The wire passed through a copper sleeve and terminated on a type *N* coaxial fitting to which a tuned crystal detector mount was attached. A reference level of output of this crystal was established, usually the reading at the center of empty waveguide. All fields were measured by adjusting a precision attenuator to bring the crystal output to this reference level. Total attenuation of the isolator is given in db with respect to the same length of empty waveguide. Transverse field distributions were taken by varying the position of the probe, usually in 0.1 inch increments across the waveguide. Unfortunately it was not possible to probe closer than about 0.1 inch from the ferrite as capacitance loading of the probe when in close proximity to the ferrite caused inaccuracies. Variation of longitudinal position was obtained by sliding the ferrite, along with its biasing magnet, past the probe.

The Full Height Isolator

The first arrangement investigated was the full height slab of Fig. 1(a). The loss sheet was a coating of Aquadag covering the full width of the flat face of the slab leaving a $\frac{1}{4}$ inch uncoated margin at each end. Thus the coated area was 4.5 inches by 0.795 inch.

A series of measurements of attenuation in the "reverse" direction over the frequency band was made at a constant applied biasing field using three different re-

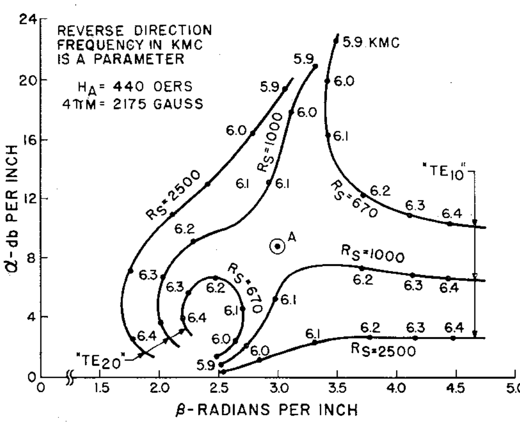

Fig. 4—Complex propagation constants of the full height isolator in the "reverse" direction with frequency and resistivity of the sheet as parameters.

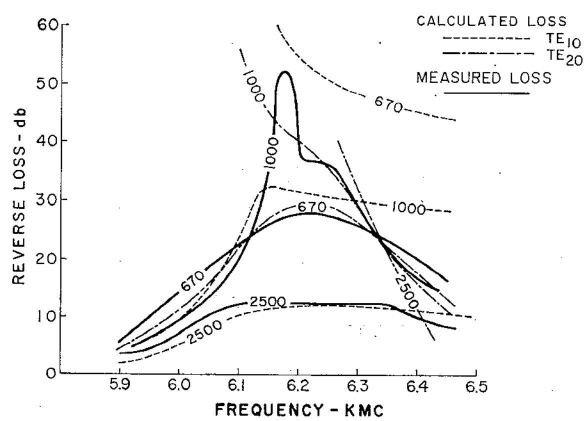

Fig. 5—Calculated and measured attenuations of the full height isolator for the "reverse" direction of transmission with sheet resistivities of 670, 1000 and 2500 ohms per square, respectively.

[10] B. Lax, K. J. Button and L. M. Roth, "Ferrite phase shifters in rectangular waveguide," *J. Appl. Phys.*, vol. 25, pp. 1413–1421; November, 1954.

[11] S. Weisbaum and H. Boyet, "Field displacement isolators for 4, 6, 11, and 24 kMc," IRE Trans. on Microwave Theory and Techniques, pp. 194–198; July, 1957.

sistivities of the loss sheet; namely, 670, 1000 and 2500 ohms per square. The results of these measurements are compared with the calculated attenuations for these cases in Fig. 5. In addition, transverse field distributions at the center of the slab are plotted along with calculated distributions in Fig. 6. Here transverse distributions were measured at both the low and high frequency ends of the band for each loss sheet. The comparisons are made on a relative shape basis, zero db being taken as the peak value in each case. In general, good agreement with the calculated distribution is obtained. With the lowest sheet resistivity, 670 ohms per square, the TE_{20} mode predominates throughout the frequency band, while with the two higher resistivities the TE_{10} mode predominates at the low frequencies and the TE_{20} mode at the high frequencies.

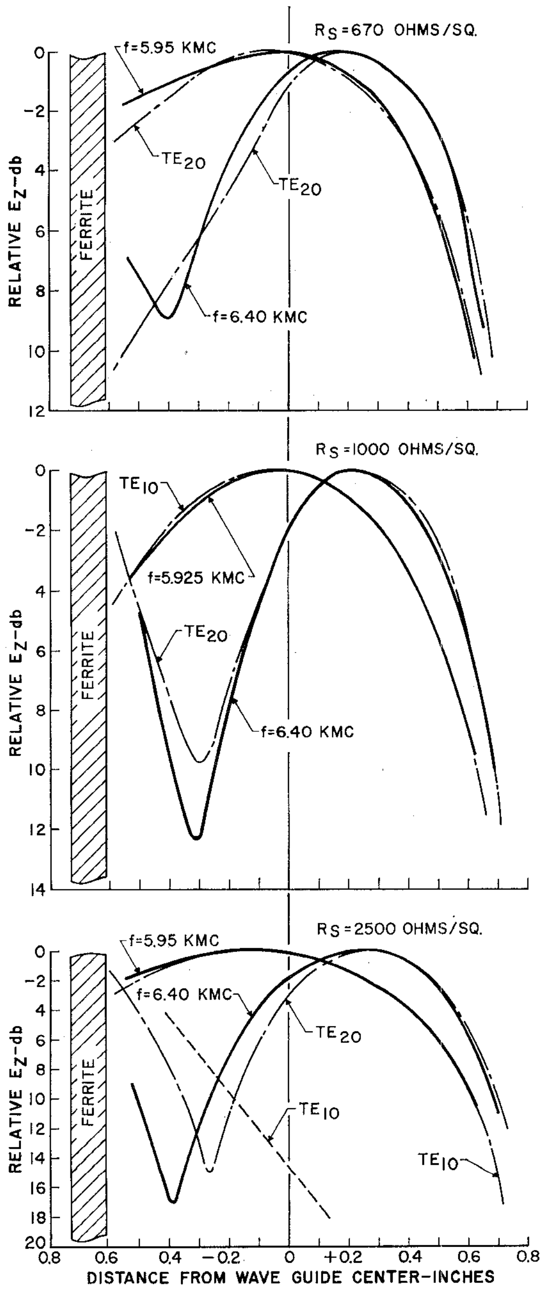

Fig. 6—Calculated and measured transverse distributions of the E field for the full height isolator with sheet resistivities of 670, 1000, and 2500 ohms per square. Solid lines indicate measured values.

Longitudinal variations of the E field are shown in Fig. 7 at the midband frequency of 6.175 kMc for the resistivity of 1000 ohms per square. Four transverse positions of the probe are represented in these curves. The curve marked 0 represents the probe at the center of the waveguide, −0.3 inch and −0.5 inch represent distance toward the ferrite and +0.3 inch represents distance away from the ferrite measured from the center of the waveguide. The signal level at the center of empty waveguide before it reaches the ferrite is the reference level, 0 db. In empty waveguide the +0.3 inch and −0.3 inch positions are normally 1.6 db down and the −0.5 inch position is 5.1 db down. In Fig. 5, we see that at this frequency the attenuations of TE_{10} and TE_{20} are comparable so that no matter which mode is favored, the attenuation will be high. There is also the possibility of an interference of fields due to each at some points in the waveguide. If one of these points comes near the center of the waveguide and near the end of the ferrite, there usually results an increased attenuation. This is probably due to a difficulty in scattering back into the TE_{10} empty waveguide mode from such a field configuration. Although this interference condition would be reactive, the wave reflected in the "forward" direction is evidently highly attenuated since no pronounced ripple in the longitudinal distribution is found.

The sharp peak of attenuation of Fig. 5 for the 1000 ohms per square case is thus explained in Fig. 7, where we see an interference which produces a low field spot at the center of the waveguide near the end of the ferrite. Without this interference we would expect an attenuation of about 31 db for this case if the TE_{10} mode carried most of the energy. The transverse distribution for the

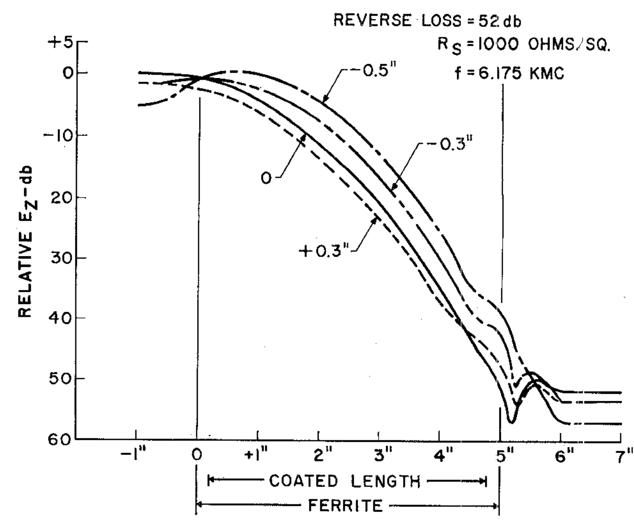

Fig. 7—Measured longitudinal distributions of the E field for the full height isolator at a frequency of 6.175 kMc for a sheet resistivity of 1000 ohms per square. Four transverse positions of the probe are used.

first four inches of the ferrite in Fig. 7 is seen to be exponential-like, that is, high field nearest the ferrite and dropping rapidly away. This is most indicative of a predominant TE_{10} mode, with the presence of the other mode being apparent only near the output end of the isolator.

The Partial Height Isolator

A partial height isolator of a design developed for a commercial application was subjected to a field probe test. The dimensional arrangement was maintained as in the regular isolator, but some mechanical details had to be changed to allow use of the probe. The attenuation of this isolator across the frequency band is shown in Fig. 8. This curve shows two peaks of attenuation. Probe measurements were made at five frequencies: one at the low frequency end of the band, 5.925 kMc; one at the first peak of attenuation, 5.958 kMc; one in the low center region at 6.10 kMc; one at the second peak 6.25 kMc; and one near the high frequency end of the band at 6.40 kMc. The results are shown on the longitudinal plots of Fig. 9.

The 5.925-kMc plot, Fig. 9(a), shows a fairly uniform attenuation with a transverse field distribution indicative of a TE_{10} type mode. The 5.958-kMc plot, Fig. 9(b), shows much the same type distribution, as might be expected since the frequency is little changed. However, at the end of the ferrite a peculiar situation exists and a sudden increase in attenuation is found. This is probably a result of an interference taking place at a position such that scatter into the normal TE_{10} mode is difficult. The 6.10-kMc plot, Fig. 9(c), shows an interference taking place before the end of the ferrite such that it has little effect on the total attenuation. The 6.25-kMc curves, Fig. 9(d), show two interferences. One in the middle of the ferrite-loaded section has little net effect, but another at the end results again in increased attenuation. The 6.40-kMc plot, Fig. 9(e), also shows two interferences, both within the ferrite-loaded portion, and the start of another at the end but the total attenuation is the least of the five cases. The first four of these field curves start out with TE_{10} type distributions at the 1-inch transverse plane for example. However, Fig. 9(e) does not show this distribution, indicating that some other mode is of magnitude comparable to TE_{10} from the beginning.

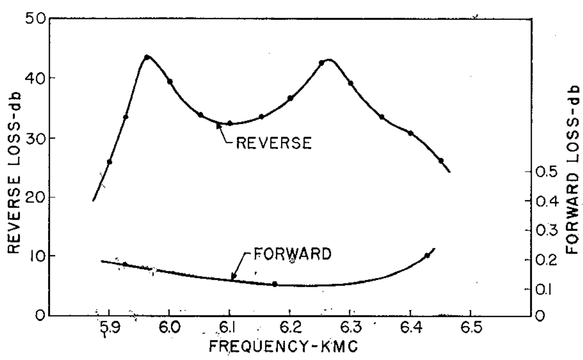

Fig. 8—Attenuation of the partial height isolator as a function of frequency.

Discussion and Conclusions

It is apparent from Fig. 4 that not much attenuation is possible at the low frequencies if both TE_{10} and TE_{20} can be excited since one of these modes will have fairly low attenuation no matter what the resistivity of the loss sheet. The transverse distributions for the low frequencies in Fig. 6 are all not markedly different from the empty waveguide TE_{10} sinusoid and hence should be

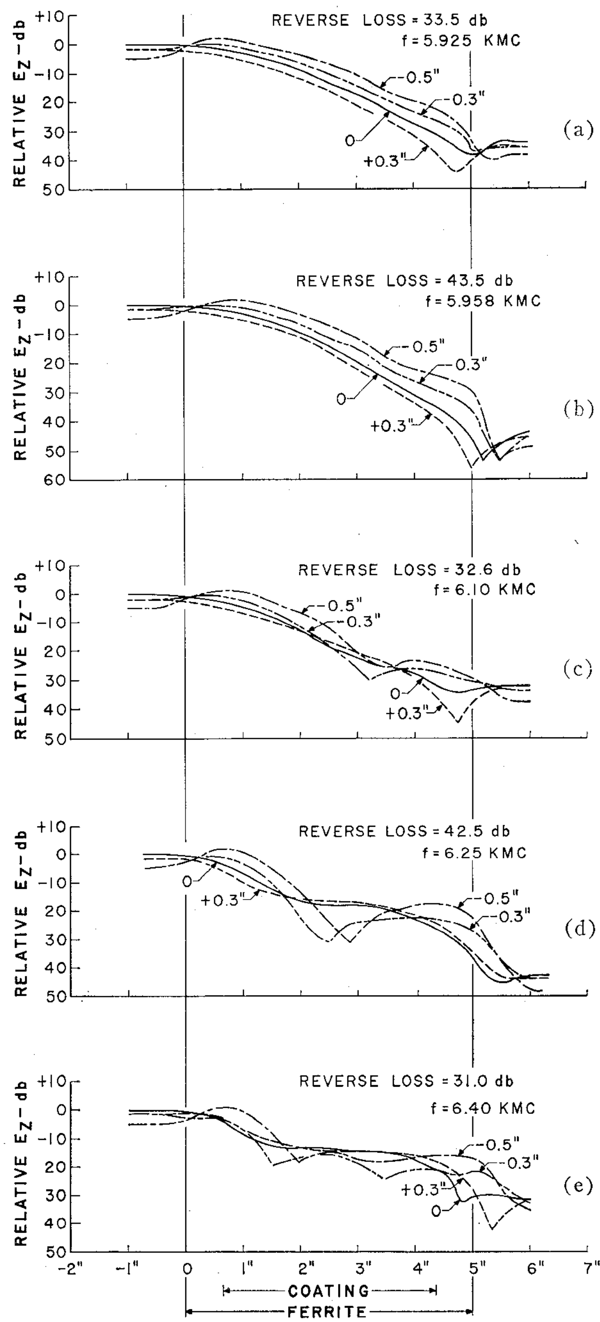

Fig. 9—Longitudinal distributions of the *E* field in the partial height isolator for the "reverse" direction of propagation at five representative frequencies. Four transverse positions of the probe are used.

easily excited. It would seem, therefore, that the most useful isolator would have its lowest frequency and resistivity corresponding to the region of point *A* in Fig. 4. Going from this point to higher frequencies, the attenuation of TE_{10} holds up well. However, that of TE_{20} drops so it appears the device is not inherently very broadband. An interference arranged to occur at a suitable frequency would help to increase the bandwidth somewhat. If some means could be found to discourage the excitation of TE_{20} without increasing the reciprocal loss appreciably, the bandwidth could be improved. The optimum resistivity of the loss sheet for the case considered here seems, from Fig. 3, to be about 900 ohms per square. The nearest to optimum performance was obtained in the case of the resistivity of 1000 ohms per square as shown in Fig. 5. Here the agreement between theory and experiment is quite good except in the region of the interference which the calculations do not take into account.

A single-mode concept of operation seems obviously inadequate to account for the observed performance. The effect of the resistance sheet in the case for optimum attenuation is a critical factor in determining the field distributions and cannot be regarded as a small perturbation of the field. The role of the interference in providing peaks of attenuation seems to be well established. This was first noted and discussed by H. Seidel.[12]

Although the partial height isolator has not been treated analytically, the behavior is sufficiently similar to that of the full height isolator that a general understanding of its operation is obtainable. Since fairly good attenuations are obtained at the low frequency end of its band it would seem that this corresponds to operation in the vicinity of the equivalent of point *A* of Fig. 4. It is also possible that the TE_{20} mode is not propagated as easily in this structure and that its attenuation does not drop so rapidly with frequency. The field configurations obtained seem consistent with the hypothesis that basically the TE_{10} mixes with enough of another mode to cause two useful interferences in its operating frequency band. Here, of course, the narrower coating of lower resistivity, approximately 150 ohms per square, is used. The effect of the peculiar changes in width of the pattern is not understood, but it is known they have an effect on the position of the interferences in the frequency response curve. The partial height of the ferrite and the resistive coating would permit z-dependent modes such as described by Seidel[12] and by Seidel and Fletcher[13] to be excited in this structure. These may indeed provide some of the attenuation. If they were responsible for the major part of the total attenuation, one would expect that the TE fields, particularly those away from the ferrite, would practically disappear near the beginning of the ferrite-loaded waveguide and then reappear suitably attenuated at the end, but this is not observed.

The combination of an analytical approach and experimental verification involving field mapping has provided an insight into the operation of a field displacement isolator in one rather narrow frequency range. We believe this has resulted in a sufficient knowledge of the device to provide a qualitative guide to the design of similar devices at other frequencies.

ACKNOWLEDGMENT

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[12] H. Seidel, "Anomalous propagation in ferrite loaded waveguide," PROC. IRE, vol. 44, pp. 1410–1414; October, 1956.

[13] H. Seidel and R. C. Fletcher, "Gyromagnetic modes in waveguide partially loaded with ferrite," *Bell Syst. Tech. J.*, vol. 38, pp. 1427–1456; November, 1959.